Presentation - Need words to clarify.

It is helpful to label points so that you can refer to them

Quiz 3 12.5, 12.6

Show work with words of explanation. (5 points each)

1) Find and equation of the plane containing points (3,-2,4), (-5,6,0) and (1,5,4)

P Q R

$V_1 = \vec{PQ} = \langle -8, 8, -4 \rangle$

$V_2 = \vec{PR} = \langle -2, 7, 0 \rangle$

$\vec{V_1} \times \vec{V_2} = \langle 28, 8, -40 \rangle$

good idea to check cross products

can use any vector parallel to $\vec{V_1} \times \vec{V_2}$

so I'll use $\vec{n} = \langle 7, 2, -10 \rangle$ (smaller #s)

plane: point $(3,-2,4)$ $\vec{n} = \langle 7, 2, -10 \rangle$

$7(x-3) + 2(y+2) - 10(z-4) = 0$

$7x + 2y - 10z + 23 = 0$

Notice: it is easy to check that P, Q, and R are on the plane.

2) Find the point where the line through (-1, -4, 5) and (3,4,-1) intersects the plane x+2y-z+ 1 = 0. Include a screen shot of a computer generated graph of the points, line and plane (rotate to show useful view).

P Q

You may use any software you like, but the command on geogebra for a line containing point (x,y,z) and direction vector (a,b,c) is

Line[(x,y,z), Vector[(a,b,c)]]

Line

point $(-1,-4,5)$

$\vec{v} = \vec{PQ} = \langle 4, 8, -6 \rangle$ (any multiple)

$\Rightarrow \begin{cases} x = -1 + 4t \\ y = -4 + 8t \\ z = 5 - 6t \end{cases}$

Intersection with plane:

$x + 2y - z + 1 = 0$

$(-1+4t) + 2(-4+8t) - (5-6t) + 1 = 0$

$26t = 13$

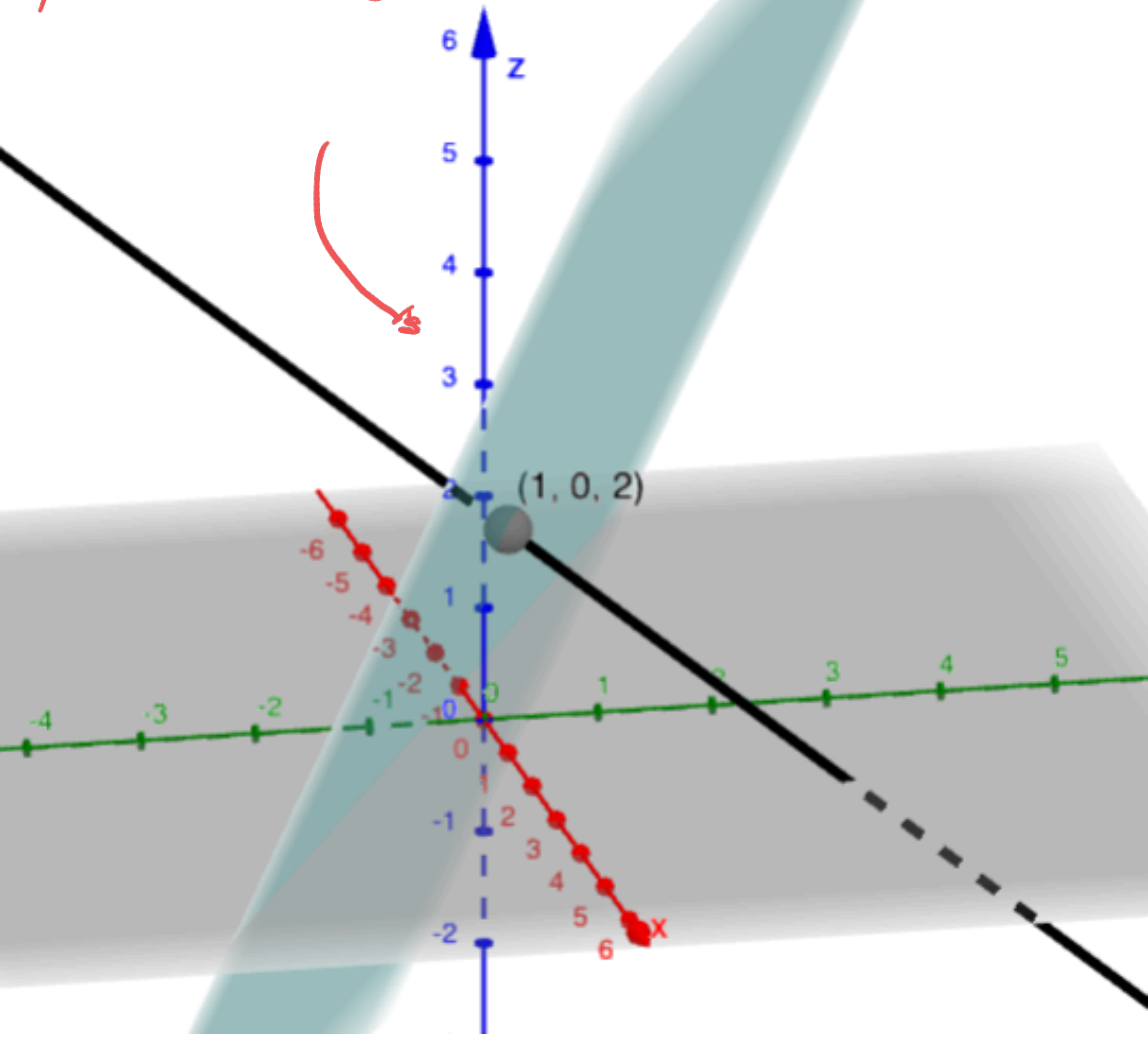
$t = 1/2$

Point of intersection

$\begin{cases} x = -1 + 4(1/2) \\ y = -4 + 8(1/2) \\ z = 5 - 6(1/2) \end{cases}$

$(1, 0, 2)$

Helpful to also plot (1,0,2) in order to check your answer



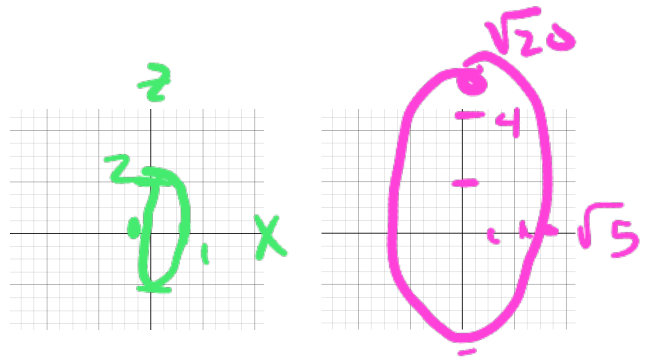
3) Sketch a graph of the following surface in R3.

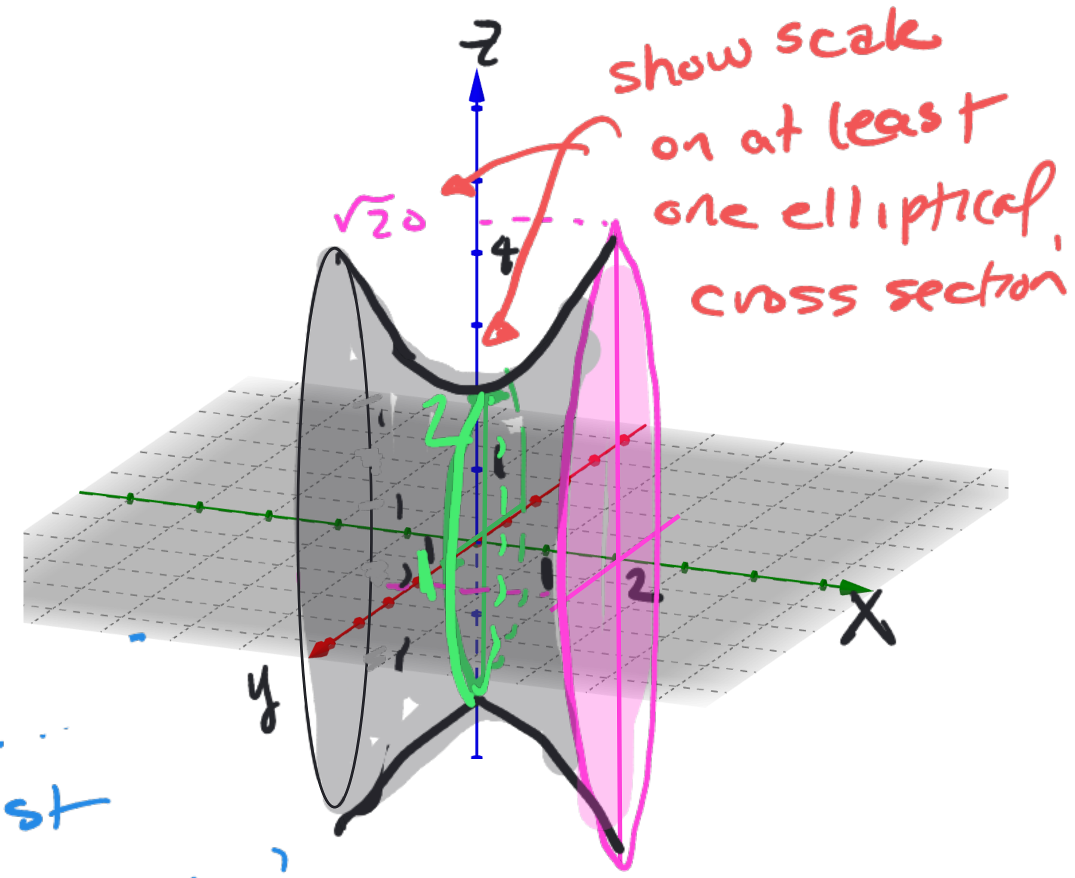
- Name the surface and give pertinent information such as traces.
  - Use small grids for traces if needed
- Show scale and label axes.

You must show an **accurate** elliptical cross section as discussed in 12.6 video 1 @ 30:40

$$x^2 - y^2 + \frac{z^2}{4} = 1$$

Name of surface: hyperboloid of one sheet





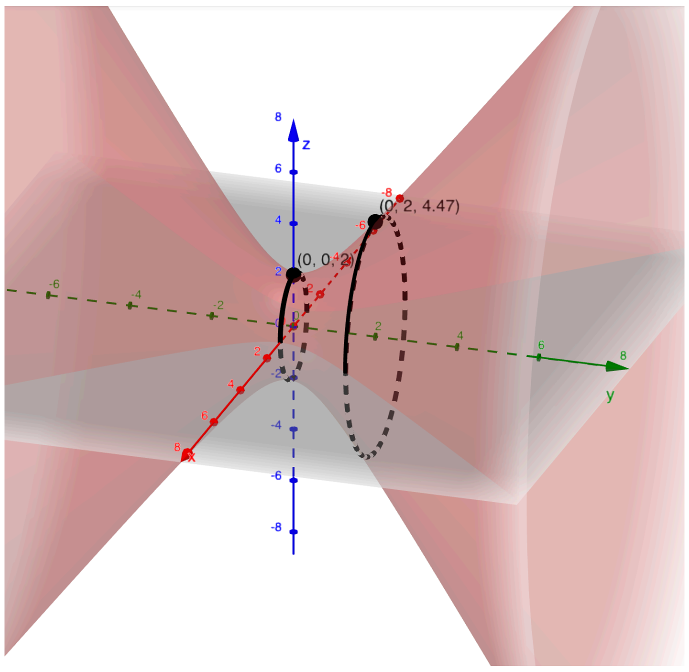
one minus sign ⇒ hyperboloid of one sheet opening in y direction

of particular interest are elliptical cross sections of the form y = K

y=0 $\quad x^2 + \frac{z^2}{4} = 1$

y=2 $\quad x^2 - 4 + \frac{z^2}{4} = 1$

$\frac{x^2}{5} + \frac{z^2}{20} = 1$



(4) The plane $x - z = 1$ and $y + 2z = 3$ intersect in a line. Find a third plane that contains this line and is perpendicular to the plane $x + y - 2z = 1$ (8 points)

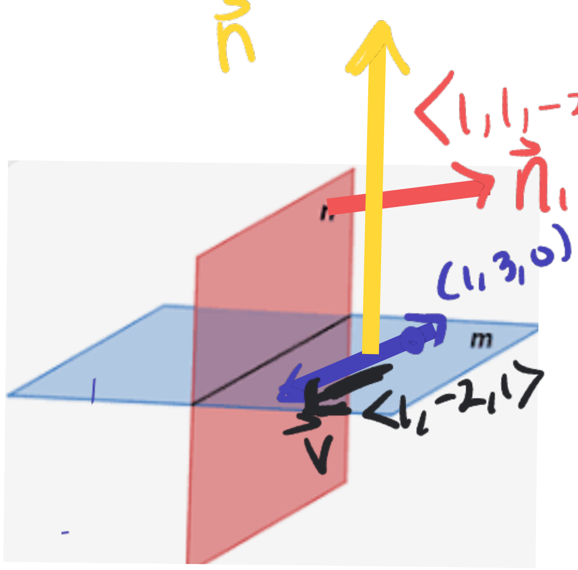
(a rough sketch or computer graph can help you visualize this)

It's a little complicated to picture the whole thing. Start by finding the line of intersection. We saw two ways of doing this.

Algebraically $\begin{cases} x - z = 1 \\ y + 2z = 3 \end{cases}$ $\quad x = z + 1 \quad y = -2z + 3 \Rightarrow \begin{cases} x = 1 + t \\ y = 3 - 2t \\ z = t \end{cases}$

line of intersect with $\vec{v} = \langle 1, -2, 1 \rangle$ P $(1, 3, 0)$

This line should be perpendicular to the plane $x + y - 2z = 1$



$\leftarrow \vec{n}_1$ is the normal vector to the given plane. our $\vec{n}$ is orthog to that

$\vec{v}$ is direction vector of the line we found

. Desired plane: need point $(1, 3, 0)$ and normal vector $\vec{n}$ must be orthog to both $\vec{v}$ and $\vec{n}_1$

so $\vec{n} = \vec{v} \times \vec{n}_1 = \begin{vmatrix} i & j & k \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{vmatrix} = \langle 3, 3, 3 \rangle$ (can use $\langle 1, 1, 1 \rangle$)

so desired plane is $x - 1 + y - 3 + z = 0$

$\boxed{x + y + z = 4}$